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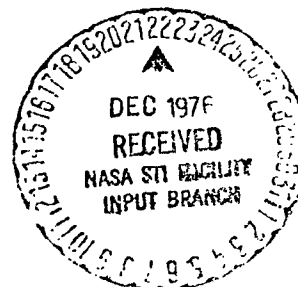
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# ORBIT MODELLING FOR SATELLITES USING THE NASA PREDICTION BULLETINS

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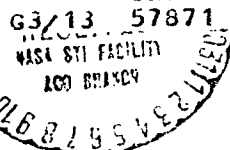


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October 1976

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ABSTRACT

For some satellites the NASA Prediction Bulletins are the only means available to the general user for obtaining orbital information. This paper provides the user with a computational interface between the information given in the NASA Prediction Bulletins and standard orbit determination programs. Such an interface is necessary to obtain accurate orbit predictions. The theoretical considerations and their computational verification in this interface modelling are presented in detail. This analysis was performed in conjunction with satellite aided search and rescue position location experiments where accurate orbits of the Amateur Satellite Corporation (AMSAT) OSCAR-6 and OSCAR-7 spacecraft are a prerequisite.

## ORBIT MODELLING FOR SATELLITES USING THE NASA PREDICTION BULLETINS

### I. INTRODUCTION

Recent feasibility experiments with the Radio Amateur Satellites OSCAR-6 and -7 indicate that a crucial factor in these experiments involving distress beacon location accuracy is the method of orbit acquisition and prediction utilized by the NASA Prediction Bulletins.\* As part of a proposed satellite aided Search and Rescue mission (SAR), the beacon latitude and longitude is to be determined by using information from individual passes of Doppler frequency shift measurements in combination with satellite orbital data (Reference 1). The OSCAR orbital data is determined by use of a simplified generalized perturbations program (SGP), in conjunction with tracking data provided by the Space Defense Center network (Reference 2). The SGP utilizes a simplified first order Kozai analytic orbit theory in order to calculate position and velocity coordinates of the OSCAR satellites.

In this paper, the expression for the semi-major axis of the orbit is rewritten so as to include the full first order short period perturbations. These effects are then introduced into the position and velocity magnitudes, thus resetting the sidereal clock, making the epoch vectors of the state commensurate with those obtained from the standard Kozai and Brou theories.

An incomplete description of the second order short period perturbation in the semi-major axis, produces no significant error in the Kozai mean

\*See "Explanation of The NASA Prediction Bulletins," distributed by Operations Center Branch, Code 512, NASA Goddard Space Flight Center, Greenbelt, Md. 20771 (February 1974).

mean-motion when the SGP orbit constants are chosen to produce a best fit to observations. However, the neglect of any terms of the first order, and often in the second order, gives a wrong instantaneous mean motion for the start of a numerical integration, which then propagates as a secular like error in time. This requirement for the semi-major axis is peculiar to the operation of using osculating elements to transfer the orbit computations to special perturbations, or even to another general perturbation.

At present, there is a need at the National Aeronautics and Space Administration for a program which will reconcile the SGP results with the approach of the more sophisticated orbit theories, thereby permitting simultaneously accurate orbit determination and distress beacon location for considerably longer prediction arcs.

Section II describes the SGP model and compares it to that of Brouwer and Kozai. Modifications to SGP are made in Section III. Sections IV and V describe the results and conclusions of calculations and comparisons of the modified SGP with the Space Defense Center SGP and the Brouwer theory.

## II. THE MODEL

Brouwer, (Reference 3) and Kozai, (Reference 4) have described the motion of an artificial Earth satellite without air drag.

In the approach of Brouwer, the equations of motion of a small mass attracted by a spheroid are written as:

$$\frac{d^2x}{dt^2} = -\frac{\partial U}{\partial x}, \quad \frac{d^2y}{dt^2} = -\frac{\partial U}{\partial y}, \quad \frac{d^2z}{dt^2} = -\frac{\partial U}{\partial z},$$

with

$$U = \frac{\mu}{r} + \frac{\mu k_2}{r^3} (1 - 3 \sin^2 \beta) + \frac{\mu k_4}{r^5} \left( 1 - 10 \sin^2 \beta + \frac{35}{3} \sin^4 \beta \right) + \dots \quad (1)$$

Here, the equatorial plane of the spheroid is taken as the Cartesian xy plane;  $\beta$  is the latitude, and if  $M$  is the mass of the spheroid and  $k$  the Gaussian constant,  $\mu = k^2 M$ . The Delaunay variables are introduced:

$$\begin{aligned} L &= (\mu a)^{1/2}, & \ell &= \text{mean anomaly,} \\ G &= L (1 - e^2)^{1/2}, & g &= \text{argument of the pericenter,} \\ H &= G \cos I, & h &= \text{longitude of ascending node,} \end{aligned} \quad (2)$$

where  $a$  and  $e$  are the osculating semi-major axis, eccentricity and inclination of the orbital plane with the orbital plane respectively. With these, the equations of motion become,

$$\frac{dL}{dt} = \frac{\partial F}{\partial \ell}, \quad \frac{d\ell}{dt} = - \frac{\partial F}{\partial L},$$

$$\frac{dG}{dt} = \frac{\partial F}{\partial g}, \quad \frac{dg}{dt} = - \frac{\partial F}{\partial G},$$

$$\frac{dH}{dt} = \frac{\partial F}{\partial h}, \quad \frac{dh}{dt} = - \frac{\partial F}{\partial H},$$

$$F = \frac{\mu^2}{2L^2} + \frac{\mu^4 k_2}{L^6} \left[ \left( -\frac{1}{2} + \frac{3}{2} \frac{H^2}{G^2} \right) \frac{a^3}{r^3} + \left( \frac{3}{2} - \frac{3}{2} \frac{H^2}{G^2} \right) \frac{a^3}{r^3} \cos(2g + 2f) \right]. \quad (3)$$

$F$  is the Hamiltonian.

The problem is solved by considering a canonical transformation from the variables  $L, G, H, \ell, g, h$  to a new set  $L', G', H', \ell', g', h'$ , and from the old to a new Hamiltonian  $F^*(L', G', H', \ell', g', h')$ . This is accomplished with the aid of a determining function  $S(L', G', H', \ell, g, h)$ , which is chosen in such a manner that  $\ell'$  is not present in  $F^*$ , while  $g'$  is permitted to appear. Consequently,  $L'$  and  $H'$  will be constants, and the system is reduced to one of one degree of freedom:

$$\frac{dG'}{dt} = \frac{\partial F^*}{\partial g'}, \quad \frac{dg'}{dt} = - \frac{\partial F^*}{\partial G'}. \quad (4)$$

After this system is solved,  $\ell'$  and  $h'$  are obtained by quadratures from

$$\frac{d\ell'}{dt} = - \frac{\partial F^*}{\partial L'}, \quad \frac{dh'}{dt} = - \frac{\partial F^*}{\partial H'}. \quad (5)$$

The determining function is obtained by a method used by von Zeipel (Reference 5), in a qualitative study of the motions of minor planets. The final equations of motion are:

$$a = a'' \left\{ 1 + \gamma_2 \left[ (-1 + 3\theta^2) \left( \frac{a''^3}{r'^3} - \eta^{-3} \right) + 3(1 - \theta^2) \frac{a''^3}{r'^3} \cos(2g' + 2f') \right] \right\}$$

$$e = e'' + \delta_1 e + \frac{\eta^2}{2e''} \left\{ \gamma_2 \left[ (-1 + 3\theta^2) \left( \frac{a''^3}{r'^3} - \eta^{-3} \right) + 3(1 - \theta^2) \left( \frac{a''^3}{r'^3} - \eta^{-4} \right) \cos(2g' + 2f') \right] - \gamma_2' (1 - \theta^2) \left[ 3e'' \cos(2g' + f') + e'' \cos(2g' + 3f') \right] \right\}$$



$$I = I'' + \delta_1 I + \frac{1}{2} \gamma_2' \theta (1 - \theta^2)^{1/2} [3 \cos (2g' + 2f') + 3e'' \cos (2g' + f') + e'' \cos (2g' + 3f')] ]$$

$$\begin{aligned} \ell = \ell' - \frac{\eta^2}{4e''} \gamma_2' \left\{ 2(-1 + 3\theta^2) \left( \frac{a''^2}{r'^2} \eta^2 + \frac{a''}{r'} + 1 \right) \sin f' \right. \\ \left. + 3(1 - \theta^2) \left[ \left( -\frac{a''^2}{r'^2} \eta^2 - \frac{a''}{r'} + 1 \right) \sin (2g' + f') \right. \right. \\ \left. \left. + \left( \frac{a''^2}{r'^2} \eta^2 + \frac{a''}{r'} + \frac{1}{3} \right) \sin (2g' + 3f') \right] \right\} \end{aligned}$$

(6)

$$\begin{aligned} g = g' + \frac{\eta^2}{4e''} \gamma_2' \left\{ 2(-1 + 3\theta^2) \left( \frac{a''^2}{r'^2} \eta^2 + \frac{a''}{r'} + 1 \right) \sin f' \right. \\ \left. + 3(1 - \theta^2) \left[ \left( -\frac{a''^2}{r'^2} \eta^2 - \frac{a''}{r'} + 1 \right) \sin (2g' + f') \right. \right. \\ \left. \left. + \left( \frac{a''^2}{r'^2} \eta^2 + \frac{a''}{r'} + \frac{1}{3} \right) \sin (2g' + 3f') \right] \right\} \\ + \frac{1}{4} \gamma_2' \left\{ 6(-1 + 5\theta^2) (f' - \ell' + e'' \sin f') + (3 - 5\theta^2) [3 \sin (2g' + 2f') \right. \\ \left. + 3e'' \sin (2g' + f') + e'' \sin (2g' + 3f')] \right\} \end{aligned}$$

$$h = h' - \frac{1}{2} \gamma_2' \theta \left[ 6 (f' - \ell' + e'' \sin f') - 3 \sin (2g' + 2f') \right. \\ \left. - 3 e'' \sin (2g' + f') - e'' \sin (2g' + 3f') \right].$$

$f', r'$  are to be computed from

$$E' - e'' \sin E' = \ell'$$

$$\tan \frac{1}{2} f' = \left( \frac{1 + e''}{1 - e''} \right)^{1/2} \tan \frac{1}{2} E'$$

$$\frac{a''}{r'} = \frac{1 + e'' \cos f'}{1 - e''^2}$$

or

$$\frac{r'}{a''} \sin f' = (1 - e''^2)^{1/2} \sin E'$$

$$\frac{r'}{a''} \cos f' = \cos E' - e''$$

$$\frac{r'}{a''} = 1 - e'' \cos E'$$

Here,  $f$  is the true anomaly,  $\eta = (1 - e''^2)^{1/2}$ ,  $\theta = \cos I''$ ,  $\gamma_2 = \frac{k_2}{a''^2}$ ,  $\gamma_2' = \gamma_2 \eta^{-4}$ , and  $k_2$  is related to the coefficient of the Earth's second zonal harmonic  $J_2$ , by the expression,

$$k_2 = J_2 \frac{R_e^2}{2}, \quad (7)$$

where  $R_e$  is the Earth's equatorial radius.

The quantities in brackets are of short period, while  $a''$ ,  $e''$ , and  $I''$  are secular in nature. Secular and long period terms of the mean anomaly, argument of perigee, and longitude of the node are contained in  $\ell'$ ,  $g'$  and  $h'$ , while long period variations in eccentricity and inclination are given by  $\delta_1 e$  and  $\delta_1 I$  respectively. The periodic terms, both long and short, are developed to  $O(k_2)$ . The secular motions are obtained to  $O(k_2^2)$ .

For the calculation of the coordinates at any time, the complete values of  $e$  and  $\ell$  should be used for the solution of Kepler's equation,

$$E - e \sin E = \ell \quad (8)$$

and subsequently, the radius vector  $r$ , and the true anomaly  $f$  which may then be used in the formulas:

$$\begin{aligned} x &= r [\cos (g + f) \cos h - \sin (g + f) \sin h \cos I] \\ y &= r [\cos (g + f) \sin h + \sin (g + f) \cos h \cos I] \\ z &= r \sin (g + f) \sin I. \end{aligned} \quad (9)$$

In a similar method by Kozai, perturbations of six orbital elements of a close Earth satellite moving in the gravitational field are obtained as functions of mean orbital elements and time. No assumptions are made about the order of magnitude of eccentricity and inclination. It is assumed however, that the density distribution of the Earth is symmetrical with respect to the axis of rotation. Also, the coefficient of the second harmonic of the potential is taken to be a small quantity of the first order, while those of the third and fourth harmonics are of the second order. In addition, the expression of the semi-major axis contains no long periodic terms.

The results include periodic perturbations of the first order, and secular perturbations up to the second order:

$$\begin{aligned}
 a &= \bar{a} + da_s, \quad \bar{a} = a_0 \left\{ 1 - \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}, \\
 e &= \bar{e} + de_s - \overline{de_s} + de_1, \\
 i &= \bar{i} + di_s - \overline{di_s} + di_1, \\
 \omega &= \omega_0 + \dot{\omega}t + d\omega_s - \overline{d\omega_s} + d\omega_1, \\
 \Omega &= \Omega_0 + \dot{\Omega}t + d\Omega_s - \overline{d\Omega_s} + d\Omega_1, \\
 M &= M_0 + \bar{n}t + dM_s, \quad \bar{n} = n_0 \left\{ 1 + \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}.
 \end{aligned} \tag{10}$$

The secular perturbations of the first order are

$$\begin{aligned}
 \bar{\omega} &= \omega_0 + \frac{A_2}{p^2} \bar{n} \left( 2 - \frac{5}{2} \sin^2 i \right) t, \\
 \bar{\Omega} &= \Omega_0 - \frac{A_2}{p^2} \bar{n} t \cos i \\
 \bar{M} &= M_0 + \bar{n} t, \\
 \bar{n} &= n_0 + \frac{A_2}{p^2} n_0 \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2},
 \end{aligned} \tag{11}$$

where  $\omega_0$ ,  $\Omega_0$  and  $M_0$  are mean values at the epoch, that is, the initial values, from which periodic perturbations have been subtracted.  $n_0$  is the unperturbed mean motion, which is related to the unperturbed semi-major axis  $a_0$  by  $n_0^2 a_0^3 = G M$ .

Kozai chose to adopt as a mean value of the semi-major axis not  $a_0$  but

$$\bar{a} = a_0 \left\{ 1 - \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}.$$

so that the following relation holds:

$$\bar{n}^2 \bar{a}^3 = G M \left\{ 1 - \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}. \quad (12)$$

The transformation from Kepler elements to Delaunay's canonical ones is:

$$\begin{cases} L = \sqrt{\mu a}, & G = \sqrt{\mu a (1 - e^2)}, & H = \sqrt{\mu a (1 - e^2)} \cos i, \\ \ell = M, & g = \omega, & h = \Omega, \end{cases} \quad (13)$$

where  $\mu = G M$ ,  $G$  is the gravitational constant and  $M$  is the mass of the Earth.  $A_2$  is taken to be of the first order of small quantities and is equal to  $3 k_2$  of Brouwer above. The quantity  $p = a(1 - e^2)$ . Also,  $\bar{e}$  and  $\bar{i}$  are mean values with respect to  $M$  and  $\omega$ , and  $\omega_0$ ,  $\Omega_0$ , and  $M_0$  are also initial values from which periodic perturbations have been subtracted.

The subscripted  $\ell$  quantities are long period perturbations, and are given by:

$$de_1 = \overline{de_s} + \frac{A_2}{pa} \frac{e \sin^2 i}{4(4-5 \sin^2 i)} \left\{ \frac{14-15 \sin^2 i}{6} - \frac{A_4}{A_2^2} \frac{18-21 \sin^2 i}{7} \right\} \cos 2\omega$$

$$+ \frac{3}{4} \frac{A_3}{A_2 a} \sin i \sin \omega,$$

$$di_1 = \overline{di_s} - \frac{A_2}{p^2} \frac{e^2 \sin 2i}{8(4-5 \sin^2 i)} \left\{ \frac{14-15 \sin^2 i}{6} - \frac{A_4}{A_2^2} \frac{18-21 \sin^2 i}{7} \right\} \cos 2\omega$$

$$- \frac{3}{4} \frac{A_3}{A_2 p} e \cos i \sin \omega,$$

$$d\Omega_1 = \overline{d\Omega_s} - \frac{A_2}{p^2} \frac{e^2 \cos i}{2(4-5 \sin^2 i)} \left[ \left\{ \frac{7-15 \sin^2 i}{6} - \frac{A_4}{A_2^2} \frac{9-21 \sin^2 i}{7} \right\} \right.$$

$$\left. + \frac{5 \sin^2 i}{2(4-5 \sin^2 i)} \left\{ \frac{14-15 \sin^2 i}{6} - \frac{A_4}{A_2^2} \frac{18-21 \sin^2 i}{7} \right\} \right] \sin 2\omega$$

$$+ \frac{3}{4} \frac{A_3}{A_2 p} \frac{\cos i}{\sin i} e \cos \omega,$$

(14)

and,

$$d\omega_1 = \overline{d\omega_s} - \frac{3}{8} \frac{A_2}{p^2} \sin^2 i \sin 2\omega$$

$$- \frac{A_2}{p^2} \left[ \frac{1}{4-5 \sin^2 i} \left\{ \frac{14-15 \sin^2 i}{24} \sin^2 i - e^2 \frac{28-158 \sin^2 i + 135 \sin^4 i}{48} \right\} \right]$$

$$\begin{aligned}
& - \frac{A_4}{A_2^2} \left( \frac{18-21 \sin^2 i}{28} \sin^2 i - e^2 \frac{35-210 \sin^2 i + 189 \sin^4 i}{56} \right) \Bigg\} \\
& - \frac{e^2 \sin^2 i (13-15 \sin^2 i)}{(4-5 \sin^2 i)^2} \left( \frac{14-15 \sin^2 i}{24} - \frac{A_4}{A_2^2} \frac{18-21 \sin^2 i}{28} \right) \Bigg] \sin 2\omega \\
& + \frac{3}{4} \frac{A_3}{A_2 p} \frac{\sin^2 i - e^2 \cos^2 i}{\sin i} \frac{1}{e} \cos \omega.
\end{aligned}$$

The barred quantities are those short periodic perturbations with respect to the mean anomaly,

$$\begin{aligned}
\overline{de_s} &= \frac{A_2}{p^2} \sin^2 i \frac{1-e^2}{6e} \overline{\cos 2v} \cos 2\omega, \\
\overline{d\omega_s} &= \frac{A_2}{p^2} \left\{ \sin^2 i \left( \frac{1}{8} + \frac{1-e^2}{6e^2} \overline{\cos 2v} \right) + \frac{1}{6} \cos^2 i \overline{\cos 2v} \right\} \sin 2\omega, \\
\overline{di_s} &= - \frac{1}{12} \frac{A_2}{p^2} \sin 2i \overline{\cos 2v} \cos 2\omega, \\
\overline{d\Omega_s} &= - \frac{1}{6} \frac{A_2}{p^2} \cos i \overline{\cos 2v} \sin 2\omega.
\end{aligned} \tag{15}$$

Here,

$$\overline{\cos j v} = \left( \frac{-e}{1 + \sqrt{1 - e^2}} \right)^j (1 + j \sqrt{1 - e^2}),$$

where  $j = 1, 2, \dots$ . The mean values of perturbations are not zero, except for the semi-major axis.

The quantities  $\dot{\Omega}$  and  $\dot{\omega}$  for the node and argument of perigee account for secular changes through the second order:

$$\begin{aligned} \dot{\Omega} = & - \frac{A_2}{p^2} \bar{n} \cos i \left[ 1 + \frac{A_2}{p^2} \left\{ \frac{3}{2} + \frac{e^2}{6} - 2 \sqrt{1 - e^2} \right. \right. \\ & \left. \left. - \sin^2 i \left( \frac{5}{3} - \frac{5}{24} e^2 - 3 \sqrt{1 - e^2} \right) \right\} \right] - \frac{A_4}{p^4} n \cos i \frac{12 - 21 \sin^2 i}{14} \left( 1 + \frac{3}{2} e^2 \right), \end{aligned}$$

and,

(16)

$$\begin{aligned} \dot{\omega} = & \frac{A_2}{p^2} \bar{n} \left( 2 - \frac{5}{2} \sin^2 i \right) \left[ 1 + \frac{A_2}{p^2} \left\{ 2 + \frac{e^2}{2} - 2 \sqrt{1 - e^2} \right. \right. \\ & \left. \left. - \sin^2 i \left( \frac{43}{24} - \frac{e^2}{48} - 3 \sqrt{1 - e^2} \right) \right\} \right] - \frac{5}{12} \frac{A_2^2}{p^4} e^2 n \cos^4 i \\ & + \frac{A_4}{p^4} n \left[ \frac{12}{7} - \frac{93}{14} \sin^2 i + \frac{21}{4} \sin^4 i + e^2 \left( \frac{27}{14} - \frac{189}{28} \sin^2 i + \frac{81}{16} \sin^4 i \right) \right] \end{aligned}$$



where  $\bar{p} = \bar{a} (1 - \bar{e}^2)$ , and  $\bar{i}$  and  $\bar{e}$  are mean values of the inclination and the eccentricity over all periods. The leading terms of these expressions account for the first order secular variations in  $\Omega$  and  $\omega$ . Both the long period and secular perturbations are taken through second order.

The short periodic terms contain terms only of the first order however.

The results for the six elements are as follows:

$$da_s = \frac{A_2}{a} \left[ \frac{2}{3} \left( 1 - \frac{3}{2} \sin^2 i \right) \left\{ \left( \frac{a}{r} \right)^3 - (1 - e^2)^{-3/2} \right\} + \left( \frac{a}{r} \right)^3 \sin^2 i \cos 2(v + \omega) \right].$$

$$de_s = \frac{1 - e^2}{e} \frac{A_2}{a^2} \left[ \frac{1}{3} \left( 1 - \frac{3}{2} \sin^2 i \right) \left\{ \left( \frac{a}{r} \right)^3 - (1 - e^2)^{-3/2} \right\} + \frac{1}{2} \left( \frac{a}{r} \right)^3 \sin^2 i \cos 2(v + \omega) \right]$$

$$- \frac{\sin^2 i}{2e} \frac{A_2}{ap} \left\{ \cos 2(v + \omega) + e \cos(v + 2\omega) + \frac{1}{3} e \cos(3v + \omega) \right\}$$

$$d\omega_s = \frac{A_2}{p^2} \left[ \left( 2 - \frac{5}{2} \sin^2 i \right) (v - M + e \sin v) + \left( 1 - \frac{3}{2} \sin^2 i \right) \left\{ \frac{1}{e} \left( 1 - \frac{1}{4} e^2 \right) \sin v + \frac{1}{2} \sin 2v + \frac{e}{12} \sin 3v \right\} - \frac{1}{e} \left\{ \frac{1}{4} \sin^2 i + \left( \frac{1}{2} - \frac{15}{16} \sin^2 i \right) e^2 \right\} \sin(v + 2\omega) \right]$$

$$\begin{aligned}
& + \frac{e}{16} \sin^2 i \sin(v - 2\omega) - \frac{1}{2} \left( 1 - \frac{5}{2} \sin^2 i \right) \sin 2(v + \omega) \\
& + \frac{1}{e} \left\{ \frac{7}{12} \sin^2 i - \frac{1}{6} \left( 1 - \frac{19}{8} \sin^2 i \right) e^2 \right\} \sin(3v + 2\omega) \\
& + \frac{3}{8} \sin^2 i \sin(4v + 2\omega) + \frac{e}{16} \sin^2 i \sin(5v + 2\omega) \Big] \\
& \qquad \qquad \qquad (17)
\end{aligned}$$

$$di_s = \frac{1}{4} \frac{A_2}{p^2} \sin 2i \left\{ \cos 2(v + \omega) + e \cos(v + 2\omega) + \frac{e}{3} \cos(3v + 2\omega) \right\},$$

$$\begin{aligned}
d\Omega_s = & - \frac{A_2}{p^2} \cos i \left\{ v - M + e \sin v - \frac{1}{2} \sin 2(v + \omega) \right. \\
& \left. - \frac{e}{2} \sin(v + 2\omega) - \frac{e}{6} \sin(3v + 2\omega) \right.
\end{aligned}$$

$$e dM_s = \frac{A_2}{p^2} \sqrt{1 - e^2} \left[ - \left( 1 - \frac{3}{2} \sin^2 i \right) \left\{ \left( 1 - \frac{e^2}{4} \right) \sin v + \frac{e}{2} \sin 2v + \frac{e^2}{12} \sin 3v \right\} \right.$$

$$+ \sin^2 i \left\{ \frac{1}{4} \left( 1 + \frac{5}{4} e^2 \right) \sin(v + 2\omega) - \frac{e^2}{16} \sin(v - 2\omega) \right.$$

$$\left. - \frac{7}{12} \left( 1 - \frac{e^2}{23} \right) \sin(v + 2\omega) - \frac{3}{8} e \sin(4v + 2\omega) - \frac{e^2}{16} \sin(5v + 2\omega) \right\} \Big]$$

With use of Kepler's law

$$M = E - e \sin E, \quad (18)$$

the relations between the true and eccentric anomaly  $v$ , and  $E$ ,

$$\cos v = \frac{\cos E - e}{1 - e \cos E},$$

and

$$\sin v = \frac{(1 - e^2)^{1/2} \sin E}{1 - e \cos E}, \quad (19)$$

and equations (10), the position of a satellite on an ellipse with a varying shape is given by,

$$r = \frac{a(1 - e^2)}{1 + e \cos v}, \quad (20)$$

and

$$\sin \delta = \sin i \cdot \sin (v + \omega).$$

Here  $(v + \omega)$  is the true longitude,  $u$ .

The SGP is a general perturbations program whose basic theory is that of Kozai. Its gravitational part is truncated to include only the effects of the first three zonal harmonics of the Earth's potential. In addition, the final equations of motion are carried through the first order accuracy in terms of the small parameter  $J_2$ , the coefficient of the Earth's second zonal harmonic. Included in the algorithm is a drag formulation which is based on the assumption that a mean element set will have its time variation given by the first few terms of a Taylor series.

The classical mean element input set includes the inclination  $i_0$ , right ascension of the ascending node  $\Omega_0$ , eccentricity  $e_0$ , argument of perigee  $\omega_0$ , mean anomaly  $M_0$ , and mean motion  $n_0$ . After updating the mean elements for first order secular and long period effects, the following quantities are calculated, which involve the position, velocity, and true latitude respectively:

$$r = a(1 - e \cos E)$$

$$\dot{r} = \frac{\sqrt{\mu a}}{r} e \sin E$$

$$r\dot{v} = \frac{\sqrt{\mu p_L}}{r}$$

$$\sin u = \frac{a}{r} \left[ \sin(E + \omega) - a_{yNSL} - a_{xNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}} \right]$$

and,

$$\cos u = \frac{a}{r} \left[ \cos(E + \omega) - a_{xNSL} + a_{yNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}} \right]. \quad (21)$$

$(E + \omega)$  is obtained from Kepler's equation by an iteration procedure,  $a$  and  $e$  include atmospheric perturbations,

$$p_L = a(1 - e_L^2)$$

$$e_L^2 = (a_{xNSL})^2 + (a_{yNSL})^2$$

$$a_{xNSL} = e \cos \omega_{S0}$$

and

$$a_{yNSL} = e \sin \omega_{s0} - \frac{1}{2} \frac{J_3}{J_2} \frac{a_e}{a(1-e^2)} \sin i_0. \quad (21a)$$

The subscript, S0, and L, denote secular and long period effects, and  $a_e$ , the magnitude of the Earth's equatorial radius.

The planetocentric distance, true latitude, ascending node, and inclination are then corrected for short-period perturbations as follows:

$$\begin{aligned} r &= \hat{r} + \frac{1}{4} J_2 \frac{a_e^2}{p_L^2} \sin^2 i_0 \cos 2u \\ u &= \hat{u} - \frac{1}{8} J_2 \frac{a_e^2}{p_L^2} (7 \cos^2 i_0 - 1) \sin 2u \\ \Omega &= \Omega_{s0} + \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \cos i_0 \sin 2u \\ i &= i_0 + \frac{3}{4} J_2 \frac{a_e^2}{p_L^2} \sin i_0 \cos i_0 \cos 2u \end{aligned} \quad (22)$$

From these quantities, the osculating position and velocity can now be computed for a specified time.

The SGP is thus a first order analytical approach which includes perturbation effects through the Earth's third zonal harmonic. Although the theory is not singular for any elliptic orbit, the position prediction is normally within  $\pm 5$  kilometers of the actual position between updated mean elements. It is this point to which we address ourselves in the next section, in order to make SGP consistent with both the Kozai and Brouwer approach.

### III. FIRST ORDER SHORT PERIOD FORM OF THE SEMI-MAJOR AXIS

Adopting the form of the force function used by Vinti, (Reference 6), the last, and dominant term of the short periodic contribution to the semi-major axis (equations 17), can be written,

$$\begin{aligned} da_s &= \frac{A_2}{a} \left[ \left( \frac{a}{r} \right)^3 \sin^2 i \right] \cdot \cos 2u \\ &= \frac{3}{2} J_2 \frac{a_e^2}{a} \left[ \left( \frac{a}{r} \right)^3 \sin^2 i \cdot \cos 2u \right]. \end{aligned}$$

For short period effects, we can use the relation given by Tisserand (Reference 7),

$$\left( \frac{a}{r} \right)^3 = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{a}{r} \right)^3 dM = (1 - e^2)^{-3/2},$$

and consequently, equations (23) and (17) become,

$$da_s = \frac{3}{2a} \frac{J_2 a_e^2}{(1 - e^2)^{3/2}} [\sin^2 i \cdot \cos 2u]. \quad (24)$$

Let us now consider the short period contribution to the planetocentric distance as given by equation (20). If as a first approximation, we assume the short period effects in the eccentricity to be small, due to the presence of the  $1/a^2$  factor, then this parameter is composed of the mean, secular, and long period effects, and let us designate this as  $e_L$ . In addition, the eccentricity can

be considered as constant over short intervals of time. We then have that

$$r_{sp} = \frac{da_s \cdot (1 - e_L^2)}{(1 + e_L \cos v)} \quad (25)$$

Over a period of revolution we have also that

$$\begin{aligned} \frac{1}{2\pi} \int_0^{2\pi} \frac{dv}{(1 + e_L \cos v)} &= \frac{1}{2\pi} \cdot 2 \cdot \left[ \int_0^{\pi} \frac{dv}{(1 + e_L \cos v)} \right] \\ &= \frac{1}{\sqrt{1 - e_L^2}} \cdot (1 > e > 0). \end{aligned} \quad (26)$$

Substituting this result and equation (24) into (25), we obtain

$$r_{sp} = \frac{3}{2} J_2 \frac{a_e^2}{p_L} \sin^2 i \cdot \cos 2u, \quad (27)$$

as the short period contribution of the semi-major axis to the planetocentric distance of the satellite. This result differs from the first equation of (22), in that the leading coefficient here is 3/2 instead of 1/4. This discrepancy, and hence that of the corresponding semi-major, produces no significant change in the SGP mean mean-motion when the orbit constants are chosen to produce a best fit to observations. However, the factor of one quarter produces an incorrect instantaneous mean motion for the start of a power series or numerical integration, and a corresponding discrepancy in the mean mean-motion results. Thus, equation (24) has the effect of resetting the sidereal clock for the satellite orbit.

To further modify the SGP, the changes in the semi-major axis and planetocentric distance are then incorporated into the expressions for  $\dot{r}$  and  $r \dot{v}$  of equations (21), by the inclusion of the short period effects in the velocity.

The velocity components are now calculated after the orbital parameters of equations (22) are determined.

Following the approach of Kozai, and paralleling equations (10) and (11) of Section II, since the mean value of the semi-major axis is taken to be

$$\bar{a} = a_0 \left\{ 1 - \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}, \quad (28)$$

then the following relation holds:

$$\bar{n}^2 \bar{a}^3 = GM \left\{ 1 - \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\},$$

from which,

$$\bar{n} = n_0 \left\{ 1 + \frac{A_2}{p^2} \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1 - e^2} \right\}. \quad (29)$$

Here,

$$a_0 = \left( \frac{GM}{n_0^2} \right)^{1/3}, \quad (30)$$

where  $n_0$  is the unperturbed or mean mean-motion provided by the two-line element input. In our modified version of the SGP, we first calculate  $a_0$  from



equation (30) above, and then utilize  $\bar{a}$  and  $\bar{n}$  from equations (28) and (29) together as starting values for the mean semi-major axis and mean motion, whereas the Space Defense Center SGP program utilizes  $\bar{a}$  from equation (26) in conjunction with the given value of the mean mean-motion,  $n_0$ .

#### IV. RESULTS

Table I is a comparison between the Space Defense Center SGP and Goddard (932) SGP with a Brouwer-Navigational Analysis Program (NAP) system involving real data from the OSCAR-7 satellite. This data is fitted with Space Defense Center SGP analytic theory and published in the NASA Prediction Bulletin number 87. The epoch of the data for bulletin 87 is, November 2, 16 hours, 47 minutes, 51.989856 seconds. The fitted orbital elements for this bulletin are as follows:

Inclination,	$i = 101.6817$ degrees
Right Ascension of the Ascending Node,	$\Omega = 351.4182$ degrees
Eccentricity,	$e = 0.0012030$
Argument of Perigee,	$\omega = 269.8988$ degrees
Mean Anomaly,	$M = 90.0660$ degrees
Mean Motion,	$n_0 = 12.53311719 \frac{\text{revolutions}}{\text{day}}$

The Space Defense Center SGP, the Brouwer and Kozai methods, as well as the Goddard 932 SGP programs employ basically the same fundamental theoretical approach to the satellite motion problem, as opposed for example, to that of Vinti (Reference 6). While both SGP programs and Brouwer and Kozai, all treat the periodic variations through first order, only the latter two account for second order accuracy in the secular motions. This contrasts to the highly

accurate, non perturbation, separable Hamiltonian approach of Vinti which carries all terms to a minimum of third order accuracy. In addition, both the mean-motion and the short period form of the semi-major axis differ between Space Defense Center and Goddard version of the SGP. In the latter program, these quantities are expressed similar to those of Kozai, and in the case of the OSCAR 6 and 7 satellites, this difference in the semi-major axis, yields an initial discrepancy of approximately eight kilometers in position between the Space Defense and Goddard 932 SGP at the epoch.

The primary differences between the Space Defense Center and Goddard 932 SGP methods can essentially be attributed then to the respective definitions of the mean-motions, the corresponding mean and osculating semi-major axes, and the resulting short period contribution to the planetocentric distance of the satellite.

Tables IA and IB show the comparison of Cartesian coordinates at the epoch of NASA Prediction Bulletin number 87. Differences in these coordinates between the Space Defense SGP, the Goddard 932 SGP, and the Brouwer analytic theory can be attributed to several causes. The Space Defense program calculations were fitted to the OSCAR 6 and 7 data. Since the two SGP programs are fundamentally similar to the theory of Brouwer, then there is sufficient reason to warrant a second order term in the short period perturbation of the semi-major major axis of the SGP version, as was done by Cohen and Lyddane (Reference 8) for the Brouwer theory.

This is presently being undertaken by the authors. In addition, there is the added sophistication of the Brouwer and Kozai methods, that is not present in

either of the SGP calculations. In the former, the secular motions are developed to second order, while in SGP, these terms are carried to first order only. The comparison after three days is somewhat more striking, with the effect of the mean-motion clearly evident. These results are listed in Tables IC and ID.

Very similar results were obtained for the NASA Prediction Bulletins 96 and 276, and these are shown in Tables II and III. Bulletins 87 and 96 are specific epochs for OSCAR-7, and bulletin 276 relates to the OSCAR-6 spacecraft.

Table IA

Comparison of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 87

	NORAD SPG	Goddard 932 SGP	Brouwer
x =	7743.753 km	7748.970 km	7748.569 km
y =	-1168.462 km	-1170.767 km	-1169.135 km
z =	-0.739 km	6.515 km	-0.979 km
$\dot{x}$ =	-0.213 km/s	-0.216 km/s	-0.213 km/s
$\dot{y}$ =	-1.428 km/s	-1.427 km/s	-1.428 km/s
$\dot{z}$ =	6.987 km/s	6.984 km/s	6.984 km/s

Table IB

Differences of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 87

	Brouwer-NORAD SGP	Brouwer-Goddard 932 SGP
$\Delta x =$	4.816 km	-0.401 km
$\Delta y =$	-0.673 km	1.632 km
$\Delta z =$	-0.240 km	-7.494 km
$\Delta \dot{x} =$	0.000 km/s	0.003 km/s
$\Delta \dot{y} =$	0.000 km/s	-0.001 km/s
$\Delta \dot{z} =$	-0.003 km/s	0.000 km/s

Table IC

Comparison of Cartesian Coordinates of the Propagated State After 3 Days  
For NORAD Bulletin Number 87

	NORAD SGP	Goddard 932 SGP	NAP*
$x =$	-2404.339 km	-1589.378 km	-1586.169 km
$y =$	-1284.918 km	-1406.800 km	-1408.339 km
$z =$	7340.679 km	7533.466 km	7539.555 km
$\dot{x} =$	-6.753 km/s	-6.950 km/s	-6.946 km/s
$\dot{y} =$	1.097 km/s	0.965 km/s	0.964 km/s
$\dot{z} =$	-2.021 km/s	-1.288 km/s	-1.284 km/s

\*These values are the result of the numerical integration by the NAP program of the Brouwer epoch vectors. These results agree with the propagated values provided by both the Vinti and Brouwer analytic orbit theories.

Table ID

Differences of Cartesian Coordinates of the Propagated State After 3 Days  
For NORAD Bulletin Number 87

	NAP-NORAD SGP	NAP-Goddard 932 SGP
$\Delta x =$	818.170 km	3.209 km
$\Delta y =$	-123.421 km	-1.539 km
$\Delta z =$	198.876 km	6.089 km
$\Delta \dot{x} =$	-0.193 km/s	0.004 km/s
$\Delta \dot{y} =$	-0.133 km/s	-0.001 km/s
$\Delta \dot{z} =$	0.737 km/s	0.004 km/s

Table IIA

Comparison of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 96

	NORAD SGP	Goddard 332 SGP	Brouwer
$x =$	5214.212 km	5220.873 km	5213.100 km
$y =$	5854.361 km	5857.842 km	5853.035 km
$z =$	-0.731 km	12.135 km	-0.509 km
$\dot{x} =$	1.069 km/s	1.065 km/s	1.070 km/s
$\dot{y} =$	-0.966 km/s	-0.970 km/s	-0.966 km/s
$\dot{z} =$	6.980 km/s	6.976 km/s	6.984 km/s



Table IIB

Differences of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 96

	Brouwer-NORAD SGP	Brouwer-Goddard 932 SGP
$\Delta x =$	-1.112 km	-7.773 km
$\Delta y =$	-1.326 km	-4.807 km
$\Delta z =$	0.222 km	-12.644 km
$\Delta \dot{x} =$	0.001 km/s	0.005 km/s
$\Delta \dot{y} =$	0.000 km/s	0.004 km/s
$\Delta \dot{z} =$	0.004 km/s	0.008 km/s

Table IIC

Comparison of Cartesian Coordinates of the Propagated State After 1 Day  
For NORAD Bulletin Number 96

	NORAD SPG	Goddard 932 SGP	NAP*
$x =$	64.828 km	276.813 km	266.782 km
$y =$	-2294.326 km	-2066.497 km	-2078.811 km
$z =$	7470.485 km	7530.580 km	7532.645 km
$\dot{x} =$	-4.783 km/s	-4.776 km/s	-4.773 km/s
$\dot{y} =$	-5.087 km/s	-5.166 km/s	-5.161 km/s
$\dot{z} =$	-1.526 km/s	-1.250 km/s	-1.256 km/s

\*These values are the result of the numerical integration by the NAP program of the Brouwer epoch vectors. These results agree with the propagated values provided by both the Vinti and Brouwer analytic orbit theories.

Table IID

Differences of Cartesian Coordinates of the Propagated State After 1 Day  
For NORAD Bulletin Number 96

	NAP-NORAD SGP	NAP-Goddard 932 SGP
$\Delta x =$	201.954 km	-10.031 km
$\Delta y =$	215.515 km	-12.314 km
$\Delta z =$	62.160 km	+2.065 km
$\Delta \dot{x} =$	0.010 km/s	0.003 km/s
$\Delta \dot{y} =$	-0.074 km/s	0.005 km/s
$\Delta \dot{z} =$	0.270 km/s	-0.006 km/s

Table IIIA

Comparison of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 276

	NORAD SGP	Goddard 932 SGP	Brouwer
$x =$	7605.185 km	7612.355 km	7600.206 km
$y =$	1892.096 km	1890.996 km	1888.018 km
$z =$	-0.727 km	12.926 km	12.713 km
$\dot{x} =$	0.338 km/s	0.332 km/s	0.333 km/s
$\dot{y} =$	-1.390 km/s	-1.390 km/s	-1.393 km/s
$\dot{z} =$	6.986 km/s	6.982 km/s	6.993 km/s

Table IIIB

Differences of Cartesian Coordinates at the Epoch  
of NORAD Bulletin Number 276

	Brouwer-NORAD SGP	Brouwer-Goddard 932 SGP
$\Delta x =$	-4.979 km	-12.149 km
$\Delta y =$	-4.078 km	-2.978 km
$\Delta z =$	13.440 km	-0.213 km
$\Delta \dot{x} =$	-0.005 km/s	+0.001 km/s
$\Delta \dot{y} =$	-0.003 km/s	-0.003 km/s
$\Delta \dot{z} =$	0.007 km/s	0.011 km/s

Table III C

Comparison of Cartesian Coordinates of the Propagated State After 6 Days  
For NORAD Bulletin Number 276

	NORAD SGP	Goddard 932 SGP	NAP*
$x =$	6828.173 km	7277.146 km	7267.588 km
$y =$	1644.624 km	2118.325 km	2113.978 km
$z =$	3464.997 km	2010.906 km	2013.454 km
$\dot{x} =$	-2.625 km/s	-1.306 km/s	-1.311 km/s
$\dot{y} =$	-2.274 km/s	-1.921 km/s	-1.924 km/s
$\dot{z} =$	6.233 km/s	6.739 km/s	6.746 km/s

\*These values are the result of the numerical integration by the NAP program of the Brouwer epoch vectors. These results agree with the propagated values provided by both the Vinti and Brouwer analytic orbit theories.

Table IIID

Differences of Cartesian Coordinates of the Propagated State After 6 Days  
For NORAD Bulletin Number 276

	NAP-NORAD SGP	NAP-Goddard 932 SGP
$\Delta x =$	439.415 km	-9.558 km
$\Delta y =$	469.354 km	-4.347 km
$\Delta z =$	-1451.543 km	2.548 km
$\Delta \dot{x} =$	1.314 km/s	-0.005 km/s
$\Delta \dot{y} =$	0.350 km/s	-0.003 km/s
$\Delta \dot{z} =$	0.513 km/s	0.007 km/s

## V. CONCLUSIONS

The inclusion of the full first order short period effects in the semi-major axis and corresponding planetocentric distance in the SGP analytic theory, gives a good instantaneous mean motion for the start of the Navigational Analysis numerical integration program. It has also been shown by Lyddane and Cohen (Reference 8), that in the case of a program such as Brouwer, the operation of using osculating elements to transfer the orbit computations to special perturbations, or even to another general perturbation, has the additional requirement for the semi-major axis to be of second order in short period terms. Hence, such a transformation would be of great value in eliminating any secular discrepancy.

Finally, the mean value of mean motion, as well as the semi-major axis are redefined in accordance with the relationship given by Kozai. The resulting modified version of the SGP is found to agree much more closely both at epoch and in the prediction mode, with the Brouwer Analytic Theory and the Navigational Analysis (numerical integration) Program.

## VI. ACKNOWLEDGEMENTS

Mr. John S. Watson of the Goddard Space Flight Center carried out some of the numerical computations. Mr. R. A. Gordon of the Goddard Space Flight Center took part in several useful discussions about the work.



Table IIIB

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$\Delta y =$	-4.078 km	-2.978 km
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For NORAD Bulletin Number 276

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$\dot{y} =$	-2.274 km/s	-1.921 km/s	-1.924 km/s
$\dot{z} =$	6.233 km/s	6.739 km/s	6.746 km/s

\*These values are the result of the numerical integration by the NAP program of the Brouwer epoch vectors. These results agree with the propagated values provided by both the Vinti and Brouwer analytic orbit theories.

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## VII. REFERENCES

1. "A Satellite Aided Search And Rescue Program," December 1975, Goddard Space Flight Center, Greenbelt, Maryland.
2. Major, Paul E., Roots, Felix R., and Lane, Max H., "Primary General Perturbation Theories Used In The Space Defense Center," Analysis Memorandum 75-2, Office of Astrodynamic Applications, Fourteenth Aerospace Force, Ent Air Force Base, Colorado, May 1975.
3. Brouwer, Dirk, "Solution of The Problem of Artificial Satellite Theory Without Drag," The Astronomical Journal, 64, No. 1274, November 1959, 378-397.
4. Kozai, Yoshihide, "The Motion of A Close Earth Satellite," The Astronomical Journal, 64, No. 1274, November 1959, 367-377.
5. von Zeipel, H., Ark. Astr. Mat. Fys., 11, No. 1, 1916.
6. Vinti, John P., "A New Method of Solution For Unretarded Satellite Orbits," J. of Res. Nat. Bureau of Standards 62B, 105, 1959.
7. Tisserand, F., Traité de Mécanique Céleste, Vol. I, 1889.
8. Lyddane, R. H., and Cohen, C. J., "Numerical Comparison Between Brouwer's Theory And Solution By Cowell's Method For The Orbit of An Artificial Satellite," The Astronomical Journal 67, April 1962, 176-177.

## TABLE CAPTIONS

Table IA.	Comparison of Cartesian Coordinates at the Epoch of NORAD Bulletin Number 87.
Table IB.	Differences of Cartesian Coordinates at the Epoch of NORAD Bulletin Number 87.
Table IC.	Comparison of Cartesian Coordinates of the Propagated State After 3 Days For NORAD Bulletin Number 87.
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Table IIID.	Differences of Cartesian Coordinates of the Propagated State After 6 Days For NORAD Bulletin Number 276.

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